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PRESSURIZED RING-REINFORCED OVAL CYLINDER - COMPARISON OF THEORY AND DTMB TESTS

by

Joseph Kempner, William P. Vafakos and Neil Nissel

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DEPARTMENT
of
AEROSPACE ENGINEERING
and
APPLIED MECHANICS

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SUMMARY

Theoretical results for the stresses occuring in a ring-reinforced oval cylindrical shell subjected to a hydrostatic pressure are compared with corresponding test results recently published by the David Taylor Model Basin. The cylinder treated is composed of two different skin thicknesses. The theoretical results are obtained by using the local skin thickness of the cylinder in solutions developed for ring-reinforced short oval cylinders of uniform thickness. Good agreement is shown to exist between the results of theory and test for all circumferential stresses and for the axial bending stresses, but not for the axial membrane stresses. It is indicated that the solution based upon using the local skin thickness can not be expected to yield accurate results for these latter stresses. However, these can be shifted towards the test results by applying a simple one-dimensional correction.

INTRODUCTION

In order to provide structural designers and analysts with a better understanding of the behavior of ring-reinforced oval cylindrical shells subjected to hydrostatic pressure, a subject about which little appears in the literature, the David Taylor Model Basin has published the results of recent model tests (Ref. 1). This note presents, for comparison, theoretical results for the ring-stiffened oval cylinder reported upon in Ref. 1. The theoretical results have been obtained by extending to a ring-reinforced oval cylinder an energy solution for short oval shells which has previously been successfully applied to both clamped (Ref. 2) and simply supported (Ref. 3) oval shells.

PROCEDURE FOR SOLUTION

In Ref. 2 an eight function energy solution, which is applicable to arbitrary edge conditions, is developed for short oval cylindrical shells of uniform skin thickness h. In that solution it is assumed that $h << r$ and that the local curvature $1/r$ of the median line of the oval cross section is given by

$$1/r = (1/r_o)[1 + \xi \cos (4\pi s/L_o)] \qquad (1)$$

where ξ is an eccentricity parameter which fixes the major-to-minor axis ratio b/a, L_o is the length of the oval cross section, $r_o = L_o/2\pi$ is the mean radius, and s is a circumferential coordinate measured from an end of the major axis ($0 \leq s \leq L_o$).

By applying the theorem of the minimum of the total potential the eight functions (of the axial coordinate x) are determined. The corresponding solution contains enough constants of integration to satisfy arbitrary edge conditions. The theoretical results of this note were obtained by determining the constants so as to satisfy the boundary conditions for a typical bay of a periodically ring-stiffened oval cylinder. This analysis included the determination of expressions for the displacements and stresses in the oval ring, which is subjected to the interaction

load between the ring and shell. The details of such ring calculations in which deep ring equations are used will be reported upon separately.

As is shown in Fig. 1, the cross section of the model tested by the David Taylor Model Basin is composed of two constant radii and two different skin thicknesses, i.e., the cross section is not of the form of Eq. 1 nor is h a constant. Neither of these geometric properties are consistent with the theory used. The fact that the model cross section does not conform to Eq. 1 does not pose any real difficulty in analysis, since the model can be very closely approximated by an appropriate choice of the parameters in Eq. 1. These were chosen so that both the model tested and its theoretical counterpart had the same values of $L_o = 84.44"$ and $b/a \approx 1.5$. These quantities were obtained by using $r_o = 13.44"$ and $\xi = 0.61$, which imply that the major and minor axes are $b = 31.60"$ and $a = 20.80"$, respectively. These values compare favorably with those consistent with Fig. 1, i.e., $b = 31.615"$ and $a = 20.981"$.

However, a major difficulty in applying the shell solution of Ref. 2 to the model of Fig. 1 does arise since that model is composed of two different skin thicknesses, whereas the solution is applicable only to a shell of uniform thickness. To circumvent this difficulty, the following artifice was used.

The solution for the model of Fig. 1 was assumed to be adequately approximated by using the local thicknesses of that model in the eight function solution developed in Ref. 2 for a shell of uniform thickness, i.e., the solution was obtained for each of two different theoretical models. Both models were assumed to have cross-sectional dimensions (discussed above), axial bay length L and reinforcing rings corresponding to Fig. 1. However, different uniform skin thicknesses were assumed for each model. These corresponded to the two different skin thicknesses of the model tested, i.e., $h = h_1 = 0.385"$ and $h = h_2 = 0.211"$. The solution for a quadrant of the reinforced cylinder of Fig. 1 was assumed to be given by the solution for $h = h_2$ in the range 0 (major axis) $\leq s \leq 9.42"$ (point of tangency) and by the solution for $h = h_1$ in the range $9.42 \leq s \leq 21.11"$ (minor axis).

DISCUSSION

The experimental points and the dashed curves (local radius of curvature solution) for the nondimensional stress σ = (dimensional stress/hydrostatic pressure) shown in Figs. 2 through 8 have been taken from Ref. 1. In these figures σ_x and σ_θ, respectively, are the nondimensional axial and circumferential stresses, and the location of the weld corresponds to the point of tangency in Fig. 1. Local radius of curvature solutions, in which solutions for oval cylinders are obtained by using the local radius of curvature of the oval in the corresponding solution for a circular cylinder, have been shown in Refs. 2 and 3 to yield good results for clamped and simply supported short oval cylinders. However, as has been conjectured in Ref. 2 and shown in Ref. 1, this type of approximate solution does not yield good results for the case of ring-reinforced oval cylinders. All solid lines in the figures are the result of using the local thickness of the ring-reinforced oval cylinder in the solution obtained for a shell of uniform thickness, as has been discussed in the section above.

Figs. 2 and 3, respectively, are plots of σ_x and σ_θ versus x for the inside and outside surfaces of the shell at the major axis. Figs. 4 and 5 are corresponding plots at the minor

axis, whereas Figs. 6 and 7 show these stresses for a quadrant of the shell at mid-bay. The abscissa θ in these latter figures is the angle which a normal to the median surface of the shell in Fig. 1 makes with the major axis. The junction of the two different shell thicknesses, i.e., the weld, occurs at $\theta = 62.1^{\circ}$. In Fig. 8 plots are shown for the circumferential flange stresses in the ring. These theoretical and experimental ring stresses are seen to be in particularly good agreement.

The solid curves in Figs. 2 through 8, which are based upon the eight function energy solution in which the local shell thickness has been used, show remarkable agreement between theory and test for the stresses σ_θ. In many instances these curves pass right through the test data. On the other hand, the corresponding theoretical stresses σ_x of Figs. 2 through 7 appear to differ from the test results by a translation, i.e., the theoretical stresses σ_x are too low at the major axis and too high at the minor axis. This effect is graphically illustrated in Figs. 9 through 11 in which the stresses σ_x have been separated into membrane and bending components, $(\sigma_x)_{membrane} = (1/2)[(\sigma_x)_{inner} + (\sigma_x)_{outer}]$ and $(\sigma_x)_{bending} = (1/2)[(\sigma_x)_{inner} - (\sigma_x)_{outer}]$. Here it is seen that the theoretical values for $(\sigma_x)_{bending}$ agree well with the test data, showing that the discrepancy between theory and test stresses σ_x is due to erroneous theoretical values

for $(\sigma_x)_{membrane}$ alone.

In this connection the following considerations indicate that the above solution, based upon using the local skin thicknesses of Fig. 1, can not be expected to yield accurate results for the stresses $(\sigma_x)_{membrane}$. When the two different local skin thicknesses of Fig. 1 are used in solutions for oval shells of uniform thickness subjected to a hydrostatic pressure two different axial contractions are obtained. The thinner shell contracts more than the thicker shell. In the tests of the shell of Fig. 1, however, the axial contraction was the same for both the thin and thick portions. If the thin and thick shells (in which the local thickness has been used) were considered as simple one-dimensional bars and forced to have the same axial contraction without changing the net end load, the thinner shell would have to be stretched, and the thick shell compressed. This would increase the axial membrane stress in the thin shell and decrease the corresponding stress in the thick shell, i.e., such a correction would shift the theoretical curves for σ_x towards the test data. However, using this simple one-dimensional procedure, in which the cross-sectional areas of the bars are assumed to be given by the cross-sectional areas of the thin and thick portions of the shell (Fig. 1), results in a correction to $(\sigma_x)_{membrane}$ of +5.8 for 0 (major axis) $\leq \theta \leq 62.1^{\circ}$

and -2.6 for $62.1^\circ \leq \theta \leq 90^\circ$ (minor axis). The correction can therefore only be considered as qualitative, since the magnitudes are only about 1/4 of those required (see Figs. 9 through 11, for example).

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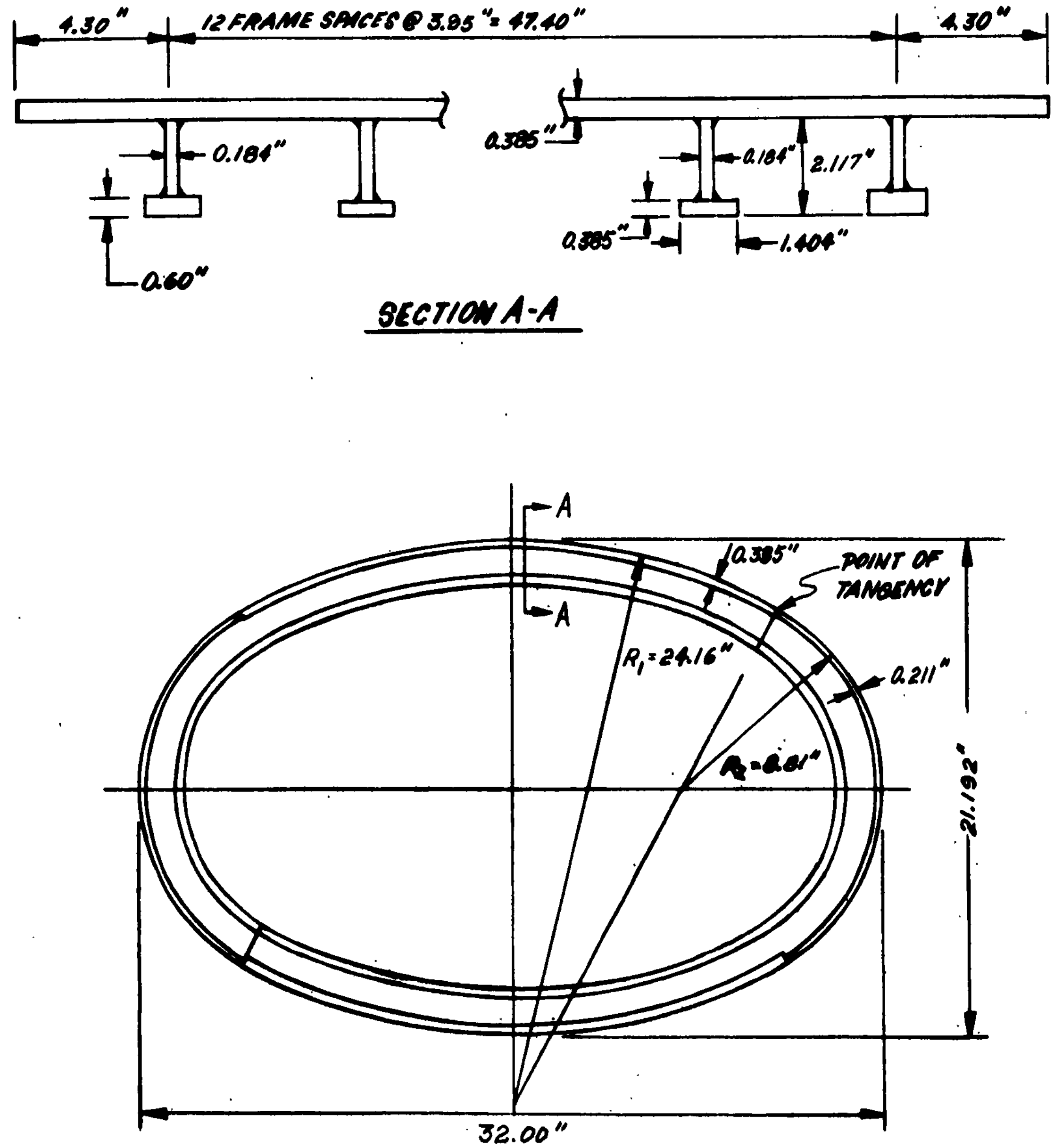


Figure 1 – Details of Model EC-1

(Reproduced from Ref. 1)

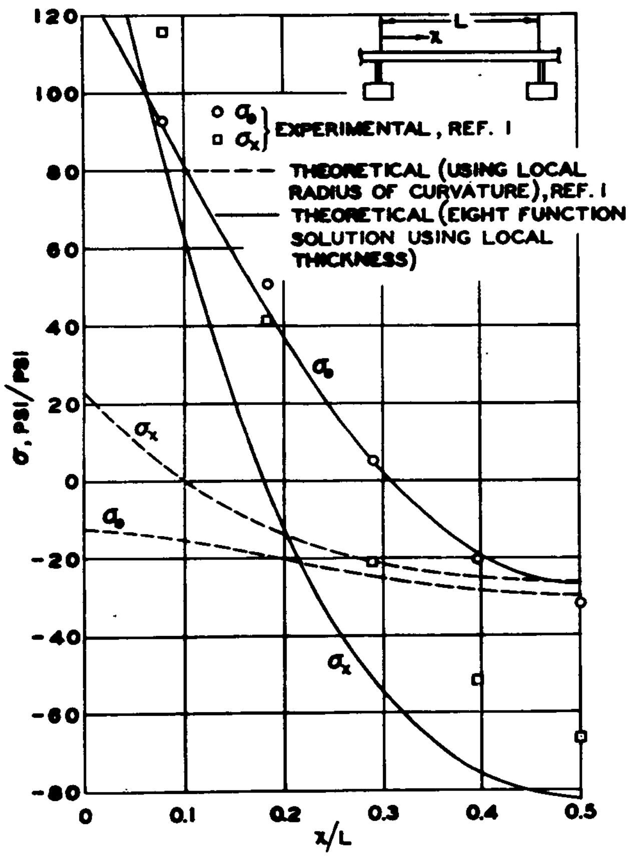


FIG. 2 STRESS DISTRIBUTION ON OUTSIDE SURFACE OF THE SHELL AT THE MAJOR AXIS

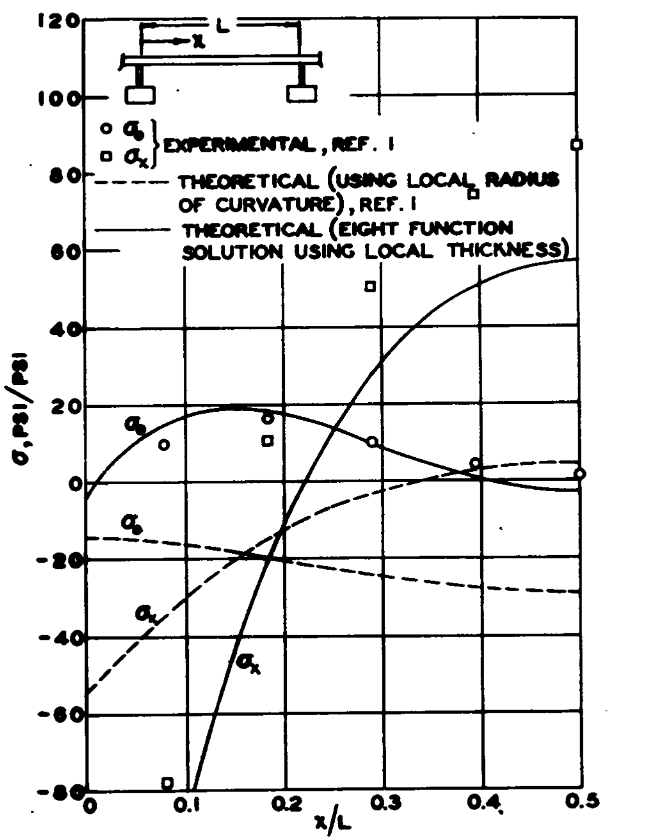


FIG. 3 STRESS DISTRIBUTION ON INSIDE SURFACE OF THE SHELL AT THE MAJOR AXIS

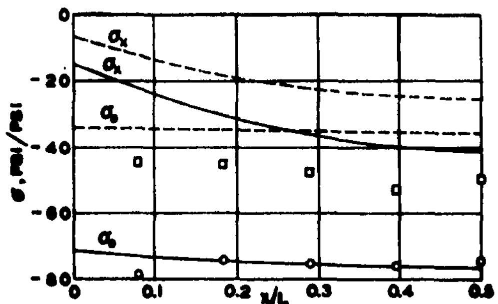
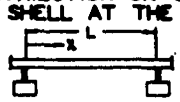


FIG. 4 STRESS DISTRIBUTION ON OUTSIDE SURFACE OF THE SHELL AT THE MINOR AXIS



○ σ_θ } EXPERIMENTAL, REF. I
□ σ_x

- - - - THEORETICAL (USING LOCAL RADIUS OF CURVATURE), REF. I

——— THEORETICAL (EIGHT FUNCTION SOLUTION USING LOCAL THICKNESS)

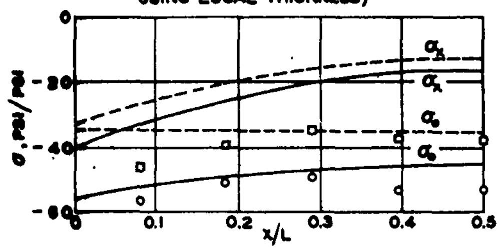


FIG. 5 STRESS DISTRIBUTION ON INSIDE SURFACE OF THE SHELL AT THE MINOR AXIS

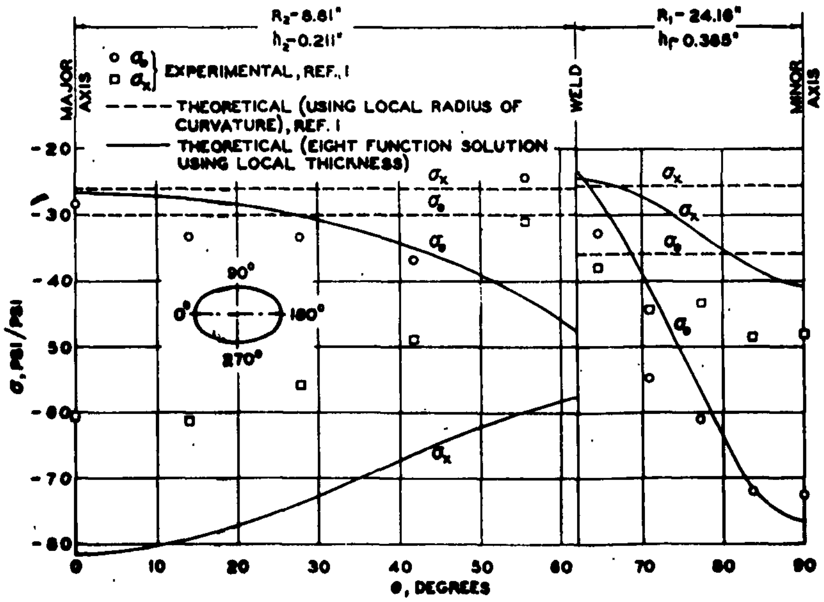


FIG. 6 MID-BAY STRESS DISTRIBUTION ON OUTSIDE SURFACE OF A QUADRANT OF SHELL

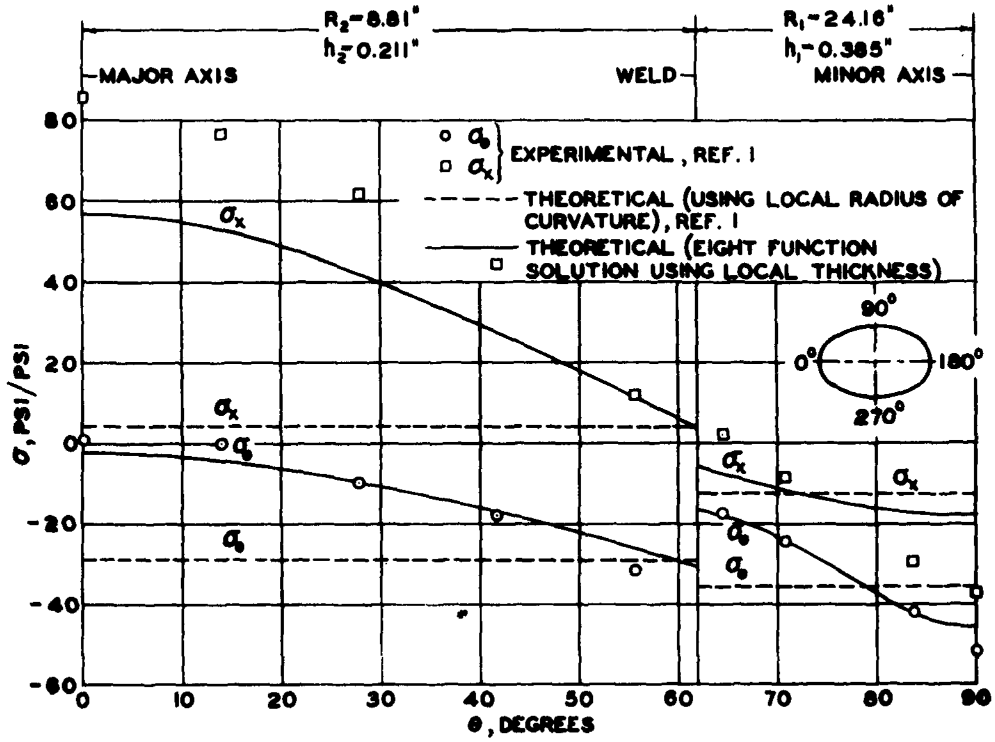


FIG. 7 MID-BAY STRESS DISTRIBUTION ON INSIDE SURFACE OF A QUADRANT OF SHELL

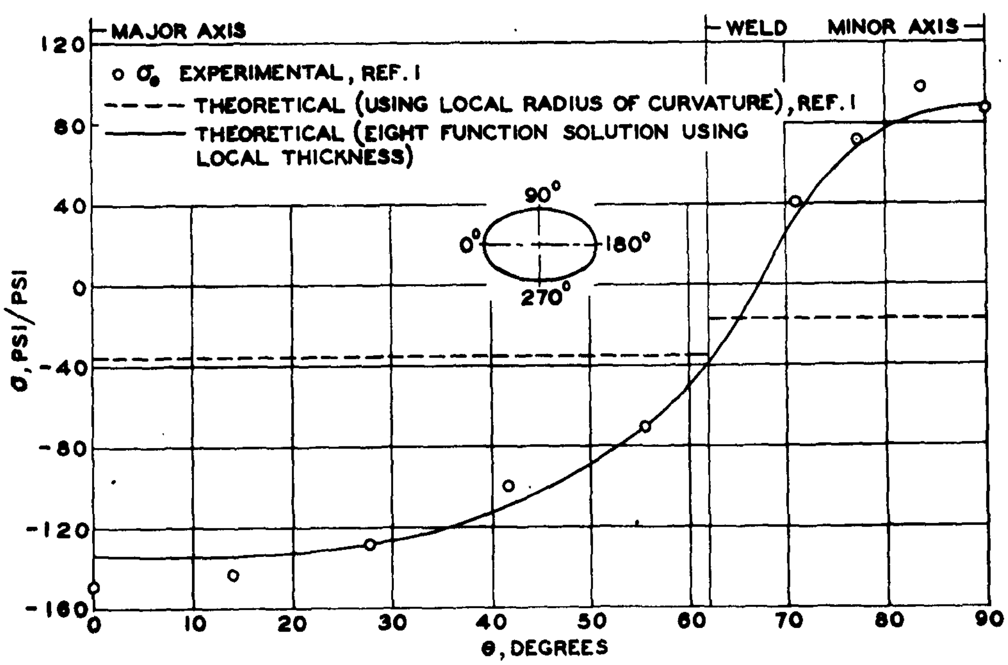


FIG. 8 CIRCUMFERENTIAL FLANGE STRESS FOR A QUADRANT OF FRAME

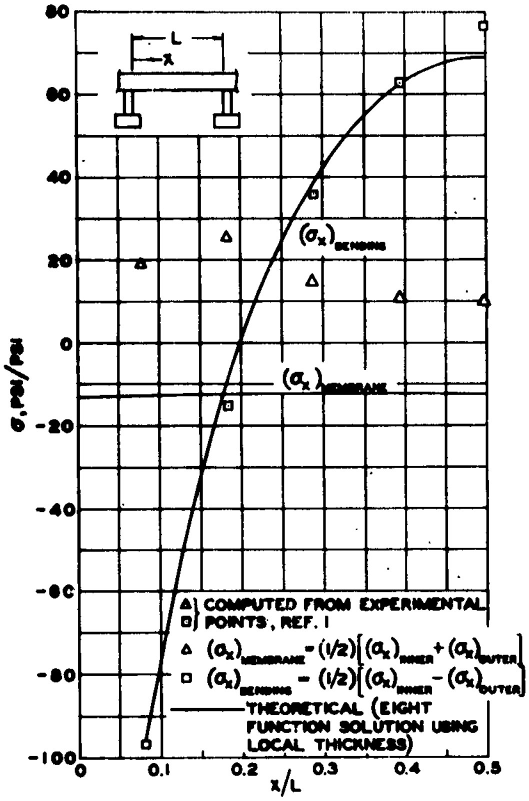


FIG. 9 AXIAL MEMBRANE AND BENDING STRESSES AT THE MAJOR AXIS

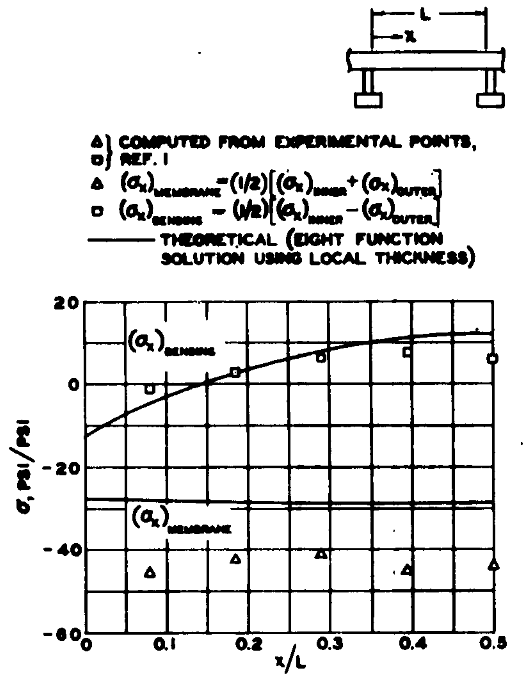


FIG. 10 AXIAL MEMBRANE AND BENDING STRESSES AT THE MINOR AXIS

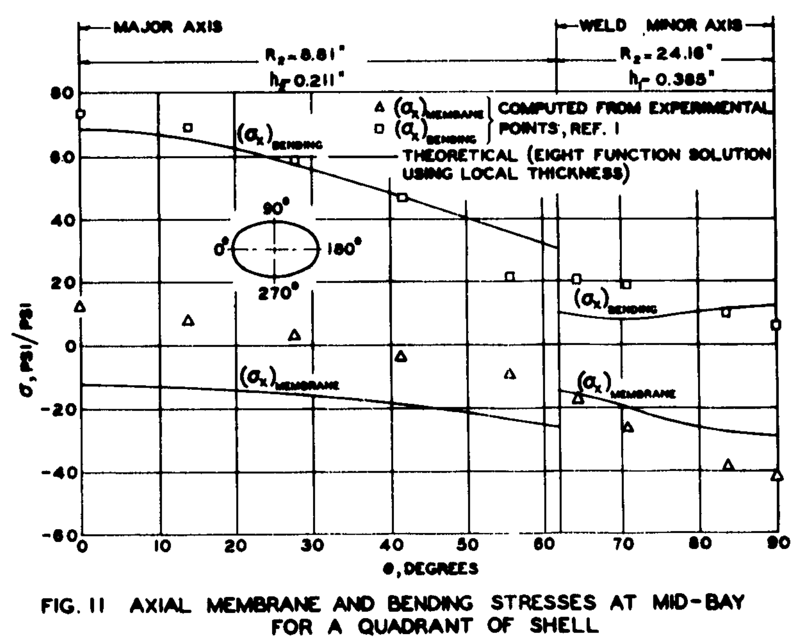


FIG. 11 AXIAL MEMBRANE AND BENDING STRESSES AT MID-BAY FOR A QUADRANT OF SHELL

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